

# 三维球坐标系下波包非线性传播过程的一种数值求解方法<sup>\*</sup>

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## 摘 要

采用全隐欧拉 (FICE) 格式, 将三维球坐标系下的大气基本运动方程作为控制方程, 建立起了三维球坐标系下可压大气中的全非线性数值模式; 对纬向边界也进行了合理的处理, 并对该模式进行了数值检验. 计算结果显示重力波波包在大气中非线性传播的三维图像: 波包整体向上传播, 波相关扰动速度随高度增加而增长.

**关键词** 三维-球坐标系- FICE 格式

## 1 引 言

在空间科学的研究过程中, 空间环境的全球变化一直是一个倍受关注的研究热点, 因为它直接影响着人类的生存和发展. 从大气动力学的角度来看, 空间环境的变化主要是指全球大气的能量与动量的传输、大气化学成分的输运. 这些过程都与全球大尺度的波动过程密切相关. 在研究全球中、高层大气动力学的过程中, 我们必须注意到: 由于地球的自转效应以及太阳辐射的影响, 高度、纬度与经度都对全球中、高层大气的结构有着重要影响.

近年来, 大气科学家和空间科学家倾向于在三维球坐标系中研究大气的各种动力学特性, 显然这使得在二维平面直角坐标系中本来就十分复杂的大气运动方程变得更加复杂, 这无疑给理论解析工作者提出了一个巨大的难题. 这是目前在研究全球空间环境变化过程中理论研究进展很慢的一个重要原因.

随着人们对空间环境变化的深入研究, 世界各国特别是发达国家越来越重视全球大气的联合观测, 这无疑是研究全球大气的一个重要而有效的手段. 但是, 这种研究手段需要巨大的经费投入, 很难保证足够多的探测设备长期联合观测. 所以从目前看来, 人们虽然积累了大量的大气观测数据, 但是这些数据还远不能全面反映全球大气大尺度波传播过程的暂态特征, 甚至还不足以建立起精度足够高的全球大气的背景风场和环流模式.

中、高层大气动力学数值研究的最终目标是建立起实用的数值模型. 很显然, 达到这一目标, 首先需要建立起球坐标下的三维数值模型, 这在计算机技术发展之前几乎是不可能的. 幸运的是近 10 年来计算机技术以惊人的速度飞速发展. 90 年代以来, 许多学者开始尝试建立三维的数值模式. 最初的数值模式是全线性或部分线性化的, 尽管这些数值模

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式有些目前仍然在广泛使用,但有关非线性效应的作用在这些数值模式中是无法反映出来的.因此,建立起三维的全非线性的数值模式是目前数值研究工作者关注的热点.三维数值模拟工作的难度主要在于:1) 寻求一种具有高精度、长稳性的格式和计算方法.从现有的三维数值模式来看,在这一点做得不是很成功,一种比较通行的办法是用谱方法<sup>[1]</sup>,这种方法实质上是将方程简化为二维方程,虽然大大简化了方程的求解过程,但同时也丢失了许多“第三维”上的非线性信息,因此,严格说来谱方法并不是真正意义上的三维模式.2) 三维模式中边界条件的处理一直是一个很棘手的问题,特别是球坐标中的两个纬向边界.在大气运动方程中,地球的南、北两极是方程的奇点,这给数值模拟工作带来了很大的困难,以往的数值模拟工作大多采用简单的处理办法绕过这一难点——计算区域中去掉地球的南、北两极<sup>[1]</sup>.这种方法显然不够完善,也不具备普遍意义.

本文从球坐标下的三维大气基本运动方程出发,初步建立起三维大气的数值模型,将二维平面直角坐标系中的 FICE 格式<sup>[2]</sup>推广到三维球坐标系下,对边界条件也进行了合理地处理,并对该模式进行了数值检验.

## 2 数值模式

### 2.1 控制方程

本文从球坐标系下的大气基本运动方程出发,忽略分子粘性,控制方程有如下形式:

$$\frac{\partial \rho}{\partial t} = -\frac{u}{r \cos \theta} \frac{\partial \rho}{\partial \varphi} - \frac{v}{r} \frac{\partial \rho}{\partial \theta} - w \frac{\partial \rho}{\partial r} - \frac{\rho}{r \cos \theta} \frac{\partial u}{\partial \varphi} - \frac{\rho}{r} \frac{\partial v}{\partial \theta} - \rho \frac{\partial w}{\partial r} - \frac{2\rho w}{r} + \frac{\rho v}{r} \tan \theta, \quad (1.1)$$

$$\begin{aligned} \frac{\partial u}{\partial t} = & -\frac{u}{r \cos \theta} \frac{\partial u}{\partial \varphi} - \frac{v}{r} \frac{\partial u}{\partial \theta} - w \frac{\partial u}{\partial r} - \frac{1}{\rho r \cos \theta} \frac{\partial P}{\partial \varphi} + \frac{uv}{r} \tan \theta - \\ & \frac{uw}{r} + 2v\Omega_e \sin \theta - 2w\Omega_e \cos \theta, \end{aligned} \quad (1.2)$$

$$\frac{\partial v}{\partial t} = -\frac{u}{r \cos \theta} \frac{\partial v}{\partial \varphi} - \frac{v}{r} \frac{\partial v}{\partial \theta} - w \frac{\partial v}{\partial r} - \frac{1}{\rho r} \frac{\partial P}{\partial \theta} - \frac{u^2}{r} \tan \theta - \frac{vw}{r} - 2u\Omega_e \sin \theta, \quad (1.3)$$

$$\frac{\partial w}{\partial t} = -\frac{u}{r \cos \theta} \frac{\partial w}{\partial \varphi} - \frac{v}{r} \frac{\partial w}{\partial \theta} - w \frac{\partial w}{\partial r} - g - \frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{u^2 + v^2}{r} + 2u\Omega_e \cos \theta, \quad (1.4)$$

$$\begin{aligned} \frac{\partial T}{\partial t} = & -\frac{u}{r \cos \theta} \frac{\partial T}{\partial \varphi} - \frac{v}{r} \frac{\partial T}{\partial \theta} - w \frac{\partial T}{\partial r} - \frac{RT}{r \cos \theta c_v} \frac{\partial u}{\partial \varphi} - \frac{RT}{rc_v} \frac{\partial v}{\partial \theta} - \frac{RT}{c_v} \frac{\partial w}{\partial r} + \\ & \frac{vRT}{rc_v} \tan \theta - \frac{2RTw}{rc_v} \tan \theta, \end{aligned} \quad (1.5)$$

$$P = R\rho T. \quad (1.6)$$

上式中  $r$  是径向(垂直)方向;  $\varphi$  ( $0^\circ \leq \varphi \leq 360^\circ$ ) 和  $\theta$  ( $-90^\circ \leq \theta \leq 90^\circ$ ) 分别为经度和纬度;  $u, v, w$  分别为纬向、经向、和垂直方向风速;  $\rho, P, T$  分别为密度、压强和温度;  $R$  为普适气体常数 ( $R = 286.9821 \text{ N}\cdot\text{m}/\text{kg}\cdot\text{K}$ );  $c_v$  为定容比热 ( $c_v = 718 \text{ J}/\text{kg}\cdot\text{K}$ );  $g$  为重力加速度 ( $g = 9.80225 \text{ m}/\text{s}^2$ );  $\Omega_e = 2.729 \times 10^{-5} \text{ rad}/\text{s}$  为地球自转角频率.

### 2.2 差分方程

首先, 将方程 (1) 改写成如下守恒形式:

$$\frac{\partial \rho}{\partial t} = -\frac{1}{r \cos \theta} \frac{\partial(\rho u)}{\partial \varphi} - \frac{1}{r} \frac{\partial(\rho v)}{\partial \theta} - \frac{\partial(\rho w)}{\partial r} - \frac{2(\rho w)}{r} + \frac{(\rho v)}{r} \tan \theta, \quad (2.1)$$

$$\begin{aligned} \frac{\partial(\rho u)}{\partial t} = & -\frac{1}{r \cos \theta} \frac{\partial(\rho u^2)}{\partial \varphi} - \frac{1}{r} \frac{\partial(\rho u v)}{\partial \theta} - \frac{\partial(\rho u w)}{\partial r} - \frac{1}{r \cos \theta} \frac{\partial P}{\partial \varphi} + \\ & \frac{2\rho u v}{r} \tan \theta - \frac{3\rho u w}{r} + 2(\rho v)\Omega_e \sin \theta - 2(\rho w)\Omega_e \cos \theta, \end{aligned} \quad (2.2)$$

$$\begin{aligned} \frac{\partial(\rho v)}{\partial t} = & -\frac{1}{r \cos \theta} \frac{\partial(\rho u v)}{\partial \varphi} - \frac{1}{r} \frac{\partial(\rho v^2)}{\partial \theta} - \frac{\partial(\rho v w)}{\partial r} - \frac{1}{r} \frac{\partial P}{\partial \theta} - \frac{\rho u^2}{r} \tan \theta + \\ & \frac{\rho v^2}{r} \tan \theta - \frac{3\rho v w}{r} - 2(\rho u)\Omega_e \sin \theta, \end{aligned} \quad (2.3)$$

$$\begin{aligned} \frac{\partial(\rho w)}{\partial t} = & -\frac{1}{r \cos \theta} \frac{\partial(\rho u w)}{\partial \varphi} - \frac{1}{r} \frac{\partial(\rho v w)}{\partial \theta} - \frac{\partial(\rho w^2)}{\partial r} - \rho g - \frac{\partial P}{\partial r} + \frac{\rho(u^2 + v^2)}{r} - \\ & \frac{2\rho w^2}{r} + \frac{\rho v w}{r} \tan \theta + 2(\rho u)\Omega_e \cos \theta, \end{aligned} \quad (2.4)$$

$$\begin{aligned} \frac{\partial T}{\partial t} = & -\frac{1}{r \cos \theta} \frac{\partial(T u)}{\partial \varphi} - \frac{1}{r} \frac{\partial(T v)}{\partial \theta} - \frac{\partial(T w)}{\partial r} + T \left(1 - \frac{R}{c_v}\right) \left(\frac{1}{r \cos \theta} \frac{\partial u}{\partial \varphi} + \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial r}\right) + \\ & \frac{v R T}{r c_v} \tan \theta - \frac{2 R T w}{r c_v} \tan \theta, \end{aligned} \quad (2.5)$$

$$P = R \rho T. \quad (2.6)$$

我们的数值计算主要是研究重力波波包的传播过程, 对于中尺度和大尺度重力波, 传播过程是一个慢变过程. 要实现这种数值模拟, 要求差分算法具有实际精度高, 长期计算稳定, 结果可靠性好的特点. 为此, 本文中的数值模拟采用 FICE<sup>[2]</sup> 格式. 为改善长期计算的稳定性和可靠性, 采用全隐式. 老时间步的值用上标  $n$  表示, 新时间步即  $(n+1)$  步的变量则一律略去上标. 在对变量进行网格分配的过程中, 为避免棋盘误差并提高计算精度, 在解域中取交错均匀网格:  $u$  在  $\varphi$  方向上的半网格点取值, 在其余方向上在整网格点上取值;  $v$  在  $\theta$  方向上的半网格点取值, 在其余方向上在整网格点上取值;  $w$  在  $r$  方向上的半网格点取值, 在其余方向上在整网格点上取值; 其余物理量均在整网格点上取值. 最后的差分方程如下.

$$\begin{aligned} \frac{1}{\Delta t}(\rho - \rho^n) = & \frac{1}{r_k \cos \theta_j \Delta \varphi} [(\rho u)_{i-\frac{1}{2},j,k} - (\rho u)_{i+\frac{1}{2},j,k}] + \frac{1}{r \Delta \theta} [(\rho v)_{i,j-\frac{1}{2},k} - (\rho v)_{i,j+\frac{1}{2},k}] + \\ & \frac{1}{\Delta r} [(\rho w)_{i,j,k-\frac{1}{2}} - (\rho w)_{i,j,k+\frac{1}{2}}] - \frac{2(\rho w)_{i,j,k}}{r_k} + \frac{(\rho v)_{i,j,k}}{r_k} \tan \theta_j, \end{aligned} \quad (3.1)$$

$$\begin{aligned} \frac{1}{\Delta t}[(\rho u)_{i+\frac{1}{2},j,k} - (\rho u)_{i+\frac{1}{2},j,k}^n] = & \frac{1}{r_k \cos \theta_j \Delta \varphi} [u_{i+\frac{1}{2},j,k} (\rho_{i,j,k} u_{i-\frac{1}{2},j,k} - \rho_{i+1,j,k} u_{i+\frac{3}{2},j,k})] + \\ & \frac{1}{r_k \Delta \theta} [(\rho u v)_{i+\frac{1}{2},j-\frac{1}{2},k} - (\rho u v)_{i+\frac{1}{2},j+\frac{1}{2},k}] + \frac{1}{\Delta r} [(\rho u w)_{i+\frac{1}{2},j,k-\frac{1}{2}} - \\ & (\rho u w)_{i+\frac{1}{2},j,k+\frac{1}{2}}] + \frac{1}{r_k \cos \theta_j \Delta \varphi} (P_{i,j,k} - P_{i+1,j,k}) + \frac{2(\rho u v)_{i+\frac{1}{2},j,k}}{r_k} \tan \theta_j - \\ & \frac{3(\rho u w)_{i+\frac{1}{2},j,k}}{r_k} + 2\Omega_e \sin \theta_j (\rho v)_{i+\frac{1}{2},j,k} - 2\Omega_e \cos \theta_j (\rho w)_{i+\frac{1}{2},j,k}, \end{aligned} \quad (3.2)$$

$$\begin{aligned} \frac{1}{\Delta t}[(\rho v)_{i,j+\frac{1}{2},k} - (\rho v)_{i,j+\frac{1}{2},k}^n] &= \frac{1}{r_k \cos \theta_{j+\frac{1}{2}} \Delta \varphi}[(\rho uv)_{i-\frac{1}{2},j+\frac{1}{2},k} - (\rho uv)_{i+\frac{1}{2},j+\frac{1}{2},k}] + \\ &\frac{1}{r_k \Delta \theta} [v_{i,j+\frac{1}{2},k} (\rho_{i,j,k} v_{i,j-\frac{1}{2},k} - \rho_{i,j+1,k} v_{i,j+\frac{3}{2},k})] + \frac{1}{\Delta r} [(\rho vw)_{i,j+\frac{1}{2},k-\frac{1}{2}} - \\ &(\rho vw)_{i,j+\frac{1}{2},k+\frac{1}{2}}] + \frac{1}{r_k \Delta \theta} (P_{i,j,k} - P_{i,j+1,k}) - \frac{(\rho u^2)_{i,j+\frac{1}{2},k}}{r_k} \tan \theta_{j+\frac{1}{2}} - \\ &\frac{(\rho v^2)_{i,j+\frac{1}{2},k}}{r_k} \tan \theta_{j+\frac{1}{2}} - \frac{3(\rho vw)_{i,j+\frac{1}{2},k}}{r_k} - 2\Omega_e \sin \theta_{j+\frac{1}{2}} (\rho u)_{i,j+\frac{1}{2},k}, \end{aligned} \quad (3.3)$$

$$\begin{aligned} \frac{1}{\Delta t}[(\rho w)_{i,j,k+\frac{1}{2}} - (\rho w)_{i,j,k+\frac{1}{2}}^n] &= \frac{1}{r_{k+\frac{1}{2}} \cos \theta_j \Delta \varphi}[(\rho uw)_{i-\frac{1}{2},j,k+\frac{1}{2}} - (\rho uw)_{i+\frac{1}{2},j,k+\frac{1}{2}}] + \\ &\frac{1}{r_{k+\frac{1}{2}} \Delta \theta} [(\rho vw)_{i,j-\frac{1}{2},k+\frac{1}{2}} - (\rho vw)_{i,j+\frac{1}{2},k+\frac{1}{2}}] + \frac{1}{\Delta r} [w_{i,j,k+\frac{1}{2}} (\rho_{i,j,k} w_{i,j,k-\frac{1}{2}} - \\ &\rho_{i,j,k+1} w_{i,j,k+\frac{3}{2}})] - \rho_{i,j,k+\frac{1}{2}} g + \frac{1}{\Delta r} (P_{i,j,k} - P_{i,j,k+1}) + \frac{(\rho u^2)_{i,j,k+\frac{1}{2}}}{r_{k+\frac{1}{2}}} + \\ &\frac{(\rho v^2)_{i,j,k+\frac{1}{2}}}{r_{k+\frac{1}{2}}} - \frac{2(\rho w^2)_{i,j,k+\frac{1}{2}}}{r_{k+\frac{1}{2}}} + \frac{(\rho vw)_{i,j,k+\frac{1}{2}}}{r_{k+\frac{1}{2}}} \tan \theta_j - 2\Omega_e \cos \theta_j (\rho u)_{i,j,k+\frac{1}{2}}, \end{aligned} \quad (3.4)$$

$$\begin{aligned} \frac{1}{\Delta t}(T_{i,j,k} - T_{i,j,k}^n) &= \frac{1}{2r_k \cos \theta_j \Delta \varphi} [u_{i-\frac{1}{2},j,k} (T_{i,j,k} + T_{i-1,j,k}) - u_{i+\frac{1}{2},j,k} (T_{i,j,k} + T_{i+1,j,k})] + \\ &\frac{1}{2r_k \Delta \theta} [v_{i,j-\frac{1}{2},k} (T_{i,j,k} + T_{i,j-1,k}) - v_{i,j+\frac{1}{2},k} (T_{i,j,k} + T_{i,j+1,k})] + \\ &\frac{1}{2\Delta r} [w_{i,j,k-\frac{1}{2}} (T_{i,j,k} + T_{i,j,k-1}) - w_{i,j,k+\frac{1}{2}} (T_{i,j,k} + T_{i,j,k+1})] + \\ &\left(1 - \frac{R}{c_v}\right) T_{i,j,k} \left[ \frac{1}{r_k \cos \theta_j \Delta \varphi} (u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}) + \frac{1}{r_k \Delta \theta} (v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}) + \right. \\ &\left. \frac{1}{\Delta r} (w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}) \right] + \frac{R(vT)_{i,j,k}}{r_k c_v} \tan \theta_j - \frac{2R(wT)_{i,j,k}}{r_k c_v}, \end{aligned} \quad (3.5)$$

$$P_{i,j,k} = R(\rho T)_{i,j,k}. \quad (3.6)$$

(3) 式中有关问题说明如下:

① 括弧的下标分配到括弧内的每个变量, 例如

$$(\rho uv)_{i+\frac{1}{2},j-\frac{1}{2},k} = \rho_{i+\frac{1}{2},j-\frac{1}{2},k} u_{i+\frac{1}{2},j-\frac{1}{2},k} v_{i+\frac{1}{2},j-\frac{1}{2},k}.$$

② 对  $\rho u^2$ 、 $\rho v^2$  和  $\rho w^2$  采用 Zip<sup>[3]</sup> 码, 即

$$(\rho u^2)_{i,j,k} = \rho_{i,j,k} u_{i+\frac{1}{2},j,k} u_{i-\frac{1}{2},j,k},$$

$$(\rho v^2)_{i,j,k} = \rho_{i,j,k} v_{i,j+\frac{1}{2},k} v_{i,j-\frac{1}{2},k},$$

$$(\rho w^2)_{i,j,k} = \rho_{i,j,k} w_{i,j,k+\frac{1}{2}} w_{i,j,k-\frac{1}{2}}.$$

③ 变量在定义点以外的值通过邻近点的值线性插值计算.

### 2.3 边界条件

#### (a) $\varphi$ 边界

在本文中  $0^\circ < \varphi < 360^\circ$ , 并取  $d\varphi = 2.5^\circ$ .  $\varphi$  边界为周期边界, 即  $U|_{\varphi=0^\circ} = U|_{\varphi=360^\circ}$ ,  $\frac{\partial U}{\partial \varphi}|_{\varphi=0^\circ} = \frac{\partial U}{\partial \varphi}|_{\varphi=360^\circ}$ , 其中  $U$  为解变量.

#### (b) 上、下边界

在本文中  $6370 \text{ km} < r < 6550 \text{ km}$ , 并取  $dr = 1.5 \text{ km}$ . 上、下边界采用近特征线边界, 由沿外行近特征线的相容关系和沿内行特征线的近特征量的过给定所组成, 关于近特征线边界的确定过程在文献 [2] 和 [4] 中已有详细介绍.

#### (c) $\theta$ 边界

在本文中  $-90^\circ < \theta < 90^\circ$ , 并取  $d\theta = 0.9^\circ$ . 由于  $\theta = \pm 90^\circ$  是控制方程的奇点, 因此  $\theta$  边界的处理一直是困扰数值模拟工作者的一个难题, 本文在  $\theta = \pm 90^\circ$  处采用直角坐标系, 以避免奇点, 具体做法如下:

以  $\theta = -90^\circ$  处为坐标原点, 建立直角坐标系, 选定  $x$  方向为  $\varphi = 0^\circ$  处的经向方向;  $y$  方向为  $\varphi = 90^\circ$  处的经向方向;  $z$  为  $r$  方向. 并令  $v_x$ 、 $v_y$ 、 $v_z$  分别为  $x$ 、 $y$ 、 $z$  三个方向的风速. 这样在南极点 ( $\theta = -90^\circ$ ) 处, 新建坐标系与原坐标系有如下近似变换关系:

$$x = |r \cos \varphi|, \quad y = |r \sin \varphi|, \quad z = r; \quad (4)$$

$$\frac{\partial U}{\partial x} = \sum_{i=1}^{N_\varphi} \frac{U_{i,2,k} - U_{i,1,k}}{x_{i,2,k}}, \quad \frac{\partial U}{\partial y} = \sum_{i=1}^{N_\varphi} \frac{U_{i,2,k} - U_{i,1,k}}{y_{i,2,k}}, \quad \frac{\partial U}{\partial z} = \frac{U_{i,1,k+1} - U_{i,1,k-1}}{2\Delta z}. \quad (5)$$

这样在直角坐标系下, 边界条件有如下形式:

$$\begin{cases} \frac{\partial v_x}{\partial t} = -v_x \frac{\partial v_x}{\partial x} - v_y \frac{\partial v_x}{\partial y} - v_z \frac{\partial v_x}{\partial z} - \frac{1}{\rho} \frac{\partial P}{\partial x}, \\ \frac{\partial v_y}{\partial t} = -v_x \frac{\partial v_y}{\partial x} - v_y \frac{\partial v_y}{\partial y} - v_z \frac{\partial v_y}{\partial z} - \frac{1}{\rho} \frac{\partial P}{\partial y}, \\ \frac{\partial v_z}{\partial t} = -v_x \frac{\partial v_z}{\partial x} - v_y \frac{\partial v_z}{\partial y} - v_z \frac{\partial v_z}{\partial z} - \frac{1}{\rho} \frac{\partial P}{\partial z} - g, \\ \frac{\partial \rho}{\partial t} = -v_x \frac{\partial \rho}{\partial x} - v_y \frac{\partial \rho}{\partial y} - v_z \frac{\partial \rho}{\partial z} - \rho \frac{\partial v_x}{\partial x} - \rho \frac{\partial v_y}{\partial y} - \rho \frac{\partial v_z}{\partial z}, \\ \frac{\partial T}{\partial t} = -\frac{\partial(Tv_x)}{\partial x} - \frac{\partial(Tv_y)}{\partial y} - \frac{\partial(Tv_z)}{\partial z} + T \left( 1 - \frac{R}{c_v} \right) \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right), \\ P = R\rho T. \end{cases} \quad (6)$$

通过计算 (7) 式求得平面直角坐标系下南极点处各物理量的值, 注意到在两个坐标系下大气的密度、温度和压强的值不变, 风速可通过下式进行转换:

$$u = 0, \quad v = v_x \cos \varphi + v_y \sin \varphi, \quad w = v_z. \quad (7)$$

通过类似的方法我们可以求解北极点处的边界条件.

### 2.4 计算方法

设上标  $n$  表示第  $n$  时间步的值 ( $n=0$  表示初态), 上标  $l$  表示第  $(n+1)$  时间步中第  $l$  次迭代步的结果. 首先, 用第  $n$  时间步的值确定第  $(n+1)$  时间步的边界值, 并且该边界值在第  $(n+1)$  时间步内点的迭代求解过程中保持不变. 在对内点的求解过程中, 假设  $U^n$  和  $U^l$  已经求得, 下面进入第  $(n+1)$  时间步中的第  $(l+1)$  次迭代, 旨在求  $U^{l+1}$ . 为此, 将原差分方程 (4) 式写成如下迭代形式:

$$\begin{aligned} \frac{1}{\Delta t}(\rho^{l+1} - \rho^n) = & \frac{1}{r_k \cos \theta_j \Delta \varphi} [(\rho u)_{i-\frac{1}{2},j,k}^{l+1} - (\rho u)_{i+\frac{1}{2},j,k}^{l+1}] + \frac{1}{r \Delta \theta} [(\rho v)_{i,j-\frac{1}{2},k}^{l+1} - \\ & (\rho v)_{i,j+\frac{1}{2},k}^{l+1}] + \frac{1}{\Delta r} [(\rho w)_{i,j,k-\frac{1}{2}}^{l+1} - (\rho w)_{i,j,k+\frac{1}{2}}^{l+1}] - \frac{2(\rho w)_{i,j,k}^l}{r_k} + \frac{(\rho v)_{i,j,k}^l}{r_k} \tan \theta_j, \end{aligned} \quad (8.1)$$

$$\begin{aligned} \frac{1}{\Delta t}[(\rho u)_{i+\frac{1}{2},j,k}^{l+1} - (\rho u)_{i+\frac{1}{2},j,k}^n] = & \frac{1}{r_k \cos \theta_j \Delta \varphi} [u_{i+\frac{1}{2},j,k}^l (\rho_{i,j,k}^l u_{i-\frac{1}{2},j,k}^l - \rho_{i+1,j,k}^l u_{i+\frac{3}{2},j,k}^l) + \\ & \frac{1}{r_k \Delta \theta} [(\rho u v)_{i+\frac{1}{2},j-\frac{1}{2},k}^l - (\rho u v)_{i+\frac{1}{2},j+\frac{1}{2},k}^l] + \frac{1}{\Delta t} [(\rho u w)_{i+\frac{1}{2},j,k-\frac{1}{2}}^l - \\ & (\rho u w)_{i+\frac{1}{2},j,k+\frac{1}{2}}^l] + \frac{1}{r_k \cos \theta_j \Delta \varphi} (P_{i,j,k}^{l+1} - P_{i+1,j,k}^{l+1}) + \frac{2(\rho u v)_{i+\frac{1}{2},j,k}^l}{r_k} \tan \theta_j - \\ & \frac{3(\rho u w)_{i+\frac{1}{2},j,k}^l}{r_k} + 2\Omega_e \sin \theta_j (\rho v)_{i+\frac{1}{2},j,k}^l - 2\Omega_e \cos \theta_j (\rho w)_{i+\frac{1}{2},j,k}^l, \end{aligned} \quad (8.2)$$

$$\begin{aligned} \frac{1}{\Delta t}[(\rho v)_{i,j+\frac{1}{2},k}^{l+1} - (\rho v)_{i,j+\frac{1}{2},k}^n] = & \frac{1}{r_k \cos \theta_{j+\frac{1}{2}} \Delta \varphi} [(\rho v u)_{i-\frac{1}{2},j+\frac{1}{2},k}^l - (\rho v u)_{i+\frac{1}{2},j+\frac{1}{2},k}^l] + \\ & \frac{1}{r_k \Delta \theta} [v_{i,j+\frac{1}{2},k}^l (\rho_{i,j,k}^l v_{i,j-\frac{1}{2},k}^l - \rho_{i,j+1,k}^l v_{i,j+\frac{3}{2},k}^l) + \frac{1}{\Delta r} [(\rho v w)_{i,j+\frac{1}{2},k-\frac{1}{2}}^l - \\ & (\rho v w)_{i,j+\frac{1}{2},k+\frac{1}{2}}^l] + \frac{1}{r_k \Delta \theta} (P_{i,j,k}^{l+1} - P_{i,j+1,k}^{l+1}) - \frac{(\rho u^2)_{i,j+\frac{1}{2},k}^l}{r_k} \tan \theta_{j+\frac{1}{2}} - \\ & \frac{(\rho v^2)_{i,j+\frac{1}{2},k}^l}{r_k} \tan \theta_{j+\frac{1}{2}} - \frac{3(\rho v w)_{i,j+\frac{1}{2},k}^l}{r_k} - 2\Omega_e \sin \theta_{j+\frac{1}{2}} (\rho u)_{i,j+\frac{1}{2},k}^l, \end{aligned} \quad (8.3)$$

$$\begin{aligned} \frac{1}{\Delta t}[(\rho w)_{i,j,k+\frac{1}{2}}^{l+1} - (\rho w)_{i,j,k+\frac{1}{2}}^n] = & \frac{1}{r_{k+\frac{1}{2}} \cos \theta_j \Delta \varphi} [(\rho u w)_{i-\frac{1}{2},j,k+\frac{1}{2}}^l - (\rho u w)_{i+\frac{1}{2},j,k+\frac{1}{2}}^l] + \\ & \frac{1}{r_{k+\frac{1}{2}} \Delta \theta} [(\rho v w)_{i,j-\frac{1}{2},k+\frac{1}{2}}^l - (\rho v w)_{i,j+\frac{1}{2},k+\frac{1}{2}}^l] + \frac{1}{\Delta r} [w_{i,j,k+\frac{1}{2}}^l (\rho_{i,j,k}^l w_{i,j,k-\frac{1}{2}}^l - \\ & \rho_{i,j,k+1}^l w_{i,j,k+\frac{3}{2}}^l) - \rho_{i,j,k+\frac{1}{2}}^{l+1} g + \frac{1}{\Delta r} (P_{i,j,k}^{l+1} - P_{i,j,k+1}^{l+1}) + \frac{(\rho u^2)_{i,j,k+\frac{1}{2}}^l}{r_{k+\frac{1}{2}}} + \\ & \frac{(\rho v^2)_{i,j,k+\frac{1}{2}}^l}{r_{k+\frac{1}{2}}} - \frac{2(\rho w^2)_{i,j,k+\frac{1}{2}}^l}{r_{k+\frac{1}{2}}} + \frac{(\rho v w)_{i,j,k+\frac{1}{2}}^l}{r_{k+\frac{1}{2}}} \tan \theta_j - 2\Omega_e \cos \theta_j (\rho u)_{i,j,k+\frac{1}{2}}^l, \end{aligned} \quad (8.4)$$

$$\frac{1}{\Delta t}(T_{i,j,k}^{l+1} - T_{i,j,k}^n) = \frac{1}{2r_k \cos \theta_j \Delta \varphi} [u_{i-\frac{1}{2},j,k}^l (T_{i,j,k}^l + T_{i-1,j,k}^l) - u_{i+\frac{1}{2},j,k}^l (T_{i,j,k}^l +$$

$$\begin{aligned}
& T_{i+1,j,k}^l] + \frac{1}{2r_k \Delta \theta} [v_{i,j-\frac{1}{2},k}^l (T_{i,j,k}^l + T_{i,j,k-1}^l) - v_{i,j+\frac{1}{2},k}^l (T_{i,j,k}^l + T_{i,j+1,k}^l)] + \\
& \frac{1}{2\Delta r} [w_{i,j,k-\frac{1}{2}}^l (T_{i,j,k}^l + T_{i,j,k-1}^l) - w_{i,j,k+\frac{1}{2}}^l (T_{i,j,k}^l + T_{i,j,k+1}^l)] + \\
& \left(1 - \frac{R}{c_v}\right) T_{i,j,k}^l \left[ \frac{1}{r_k \cos \theta_j \Delta \varphi} (u_{i+\frac{1}{2},j,k}^l - u_{i-\frac{1}{2},j,k}^l) + \frac{1}{r_k \Delta \theta} (v_{i,j+\frac{1}{2},k}^l - v_{i,j-\frac{1}{2},k}^l) + \right. \\
& \left. \frac{1}{\Delta r} (w_{i,j,k+\frac{1}{2}}^l - w_{i,j,k-\frac{1}{2}}^l) \right] + \frac{R(vT)_{i,j,k}^l}{r_k c_v} \tan \theta_j - \frac{2R(wT)_{i,j,k}^l}{r_k c_v}, \quad (8.5)
\end{aligned}$$

$$P_{i,j,k}^{l+1} = R(\rho T)_{i,j,k}^{l+1}. \quad (8.6)$$

上式中,  $l=0$  时,  $U^l$  用  $U^n$  代替, 即第  $(n+1)$  时间步中的第 1 次迭代时, 将  $U^n$  作为迭代初值. 首先由 (8.5) 式计算  $T_{i,j,k}^{l+1}$ , 然后将 (8.2)—(8.4) 和 (8.6) 代入 (8.1), 消去其中的  $(\rho u)^{l+1}$ 、 $(\rho v)^{l+1}$ 、 $(\rho w)^{l+1}$  和  $P^{l+1}$ , 求出密度方程:

$$\begin{aligned}
& -\frac{\Delta t^2 R T_{i-1,j,k}^{l+1}}{r_k^2 \cos^2 \theta_j \Delta \varphi^2} \rho_{i-1,j,k}^{l+1} - \frac{\Delta t^2 R T_{i,j-1,k}^{l+1}}{r_k^2 \Delta \theta^2} \rho_{i,j-1,k}^{l+1} + \left( \frac{\Delta t^2 g}{2\Delta r} - \frac{\Delta t^2 R T_{i,j,k-1}^{l+1}}{\Delta r^2} \right) \rho_{i,j,k-1}^{l+1} + \\
& \left( 1 + \frac{2R\Delta t^2 T_{i,j,k}^{l+1}}{r_k^2 \cos^2 \theta_j \Delta \varphi^2} + \frac{2R\Delta t^2 T_{i,j,k}^{l+1}}{r_k^2 \Delta \theta^2} + \frac{2R\Delta t^2 T_{i,j,k}^{l+1}}{\Delta z^2} \right) \rho_{i,j,k}^{l+1} - \frac{\Delta t^2 R T_{i+1,j,k}^{l+1}}{r_k^2 \cos^2 \theta_j \Delta \varphi^2} \rho_{i+1,j,k}^{l+1} - \\
& \frac{\Delta t^2 R T_{i,j+1,k}^{l+1}}{r_k^2 \Delta \theta^2} \rho_{i,j+1,k}^{l+1} - \left( \frac{\Delta t^2 g}{2\Delta r} + \frac{\Delta t^2 R T_{i,j,k+1}^{l+1}}{\Delta r^2} \right) \rho_{i,j,k+1}^{l+1} = d_{i,j,k}^n + d_{i,j,k}^l. \quad (9)
\end{aligned}$$

式中,

$$\begin{aligned}
d_{i,j,k}^n &= \rho_{i,j,k}^n + \frac{\Delta t}{r_k \cos \theta_j \Delta \varphi} [(\rho u)_{i-\frac{1}{2},j,k}^n - (\rho u)_{i+\frac{1}{2},j,k}^n] + \\
& \frac{\Delta t}{r_k \Delta \theta} [(\rho v)_{i,j-\frac{1}{2},k}^n - (\rho v)_{i,j+\frac{1}{2},k}^n] + \frac{\Delta t}{\Delta r} [(\rho w)_{i,j,k-\frac{1}{2}}^n - (\rho w)_{i,j,k+\frac{1}{2}}^n] + \\
& \frac{\Delta t^2}{r_k^2 \cos^2 \theta_j \Delta \varphi^2} [u_{i-\frac{1}{2},j,k}^l u_{i-\frac{3}{2},j,k}^l \rho_{i-1,j,k}^l - 2u_{i-\frac{1}{2},j,k}^l u_{i+\frac{1}{2},j,k}^l \rho_{i,j,k}^l + \\
& u_{i+\frac{1}{2},j,k}^l u_{i+\frac{3}{2},j,k}^l \rho_{i+1,j,k}^l] + \frac{\Delta t^2}{r_k^2 \Delta \theta^2} [v_{i,j-\frac{1}{2},k}^l v_{i,j-\frac{3}{2},k}^l \rho_{i,j-1,k}^l - \\
& 2v_{i,j-\frac{1}{2},k}^l v_{i,j+\frac{1}{2},k}^l \rho_{i,j,k}^l + v_{i,j+\frac{1}{2},k}^l v_{i,j+\frac{3}{2},k}^l \rho_{i,j+1,k}^l] + \\
& \frac{\Delta t^2}{\Delta r^2} [w_{i,j,k-\frac{1}{2}}^l w_{i,j,k-\frac{3}{2}}^l \rho_{i,j,k-1}^l - 2w_{i,j,k-\frac{1}{2}}^l w_{i,j,k+\frac{1}{2}}^l \rho_{i,j,k}^l + \\
& w_{i,j,k+\frac{1}{2}}^l w_{i,j,k+\frac{3}{2}}^l \rho_{i,j,k+1}^l] + \frac{\Delta t^2}{r_k^2 \Delta \theta \Delta \varphi} \left( \frac{1}{\cos \theta_j} + \frac{1}{\cos \theta_{j-\frac{1}{2}}} \right) [(\rho uv)_{i-\frac{1}{2},j-\frac{1}{2},k}^l - \\
& (\rho uv)_{i+\frac{1}{2},j-\frac{1}{2},k}^l] - \frac{\Delta t^2}{r_k^2 \Delta \theta \Delta \varphi} \left( \frac{1}{\cos \theta_j} + \frac{1}{\cos \theta_{j+\frac{1}{2}}} \right) [(\rho uv)_{i-\frac{1}{2},j+\frac{1}{2},k}^l - \\
& (\rho uv)_{i+\frac{1}{2},j+\frac{1}{2},k}^l] + \frac{\Delta t^2}{\Delta r \cos \theta_j \Delta \varphi} \left( \frac{1}{r_k} + \frac{1}{r_{k-\frac{1}{2}}} \right) [(\rho uw)_{i-\frac{1}{2},j,k-\frac{1}{2}}^l - (\rho uw)_{i+\frac{1}{2},j,k+\frac{1}{2}}^l] + \\
& \frac{\Delta t^2}{\Delta r \cos \theta_j \Delta \varphi} \left( \frac{1}{r_k} + \frac{1}{r_{k+\frac{1}{2}}} \right) [(\rho uw)_{i-\frac{1}{2},j,k+\frac{1}{2}}^l - (\rho uw)_{i+\frac{1}{2},j,k+\frac{1}{2}}^l] +
\end{aligned}$$

$$\begin{aligned} & \frac{\Delta t^2}{\Delta \theta \Delta r} \left( \frac{1}{r_k} + \frac{1}{r_{k-\frac{1}{2}}} \right) [(\rho v w)_{i,j-\frac{1}{2},k-\frac{1}{2}}^l - (\rho v w)_{i,j+\frac{1}{2},k-\frac{1}{2}}^l] - \\ & \frac{\Delta t^2}{\Delta \theta \Delta r} \left( \frac{1}{r_k} + \frac{1}{r_{k-\frac{1}{2}}} \right) [(\rho v w)_{i,j-\frac{1}{2},k+\frac{1}{2}}^l - (\rho v w)_{i,j+\frac{1}{2},k+\frac{1}{2}}^l], \end{aligned} \quad (10)$$

$$\begin{aligned} d_{i,j,k}^l = & -\frac{2\Delta t}{r_k} (\rho w)_{i,j,k}^l + \frac{\Delta t}{r_k} (\rho v)_{i,j,k}^l \tan \theta_j + \frac{2\Delta t^2 \tan \theta_j}{r_k^2 \cos \theta_j \Delta \varphi} [(\rho u v)_{i-\frac{1}{2},j,k}^l - (\rho u v)_{i+\frac{1}{2},j,k}^l] + \\ & \frac{3\Delta t^2}{r_k^2 \cos \theta_j \Delta \varphi} [(\rho u w)_{i-\frac{1}{2},j,k}^l - (\rho u w)_{i+\frac{1}{2},j,k}^l] + \frac{2\Omega_e \tan \theta_j \Delta t^2}{r_k \Delta \varphi} [(\rho v)_{i-\frac{1}{2},j,k}^l - \\ & (\rho v)_{i+\frac{1}{2},j,k}^l] - \frac{2\Omega_e \Delta t^2}{r_k \Delta \varphi} [(\rho w)_{i-\frac{1}{2},j,k}^l - (\rho w)_{i+\frac{1}{2},j,k}^l] + \\ & \frac{\Delta t^2}{r_k^2 \Delta \theta} (\tan \theta_{j+\frac{1}{2}} \rho_{i,j+\frac{1}{2},k}^l u_{i-\frac{1}{2},j+\frac{1}{2},k}^l u_{i+\frac{1}{2},j+\frac{1}{2},k}^l - \\ & \tan \theta_{j-\frac{1}{2}} \rho_{i,j-\frac{1}{2},k}^l u_{i-\frac{1}{2},j-\frac{1}{2},k}^l u_{i+\frac{1}{2},j-\frac{1}{2},k}^l) + \\ & \frac{\Delta t^2}{r_k^2 \Delta \theta} v_{i,j,k}^l (\tan \theta_{j+\frac{1}{2}} \rho_{i,j+\frac{1}{2},k}^l v_{i,j+1,k}^l - \tan \theta_{j-\frac{1}{2}} \rho_{i,j-\frac{1}{2},k}^l v_{i,j-1,k}^l) + \\ & \frac{3\Delta t^2}{r_k^2 \Delta \theta} [(\rho v w)_{i,j+\frac{1}{2},k}^l - (\rho v w)_{i,j-\frac{1}{2},k}^l] + \\ & \frac{2\Omega_e \Delta t^2}{r_k \Delta \theta} [\sin \theta_{i,j+\frac{1}{2},k} (\rho u)_{i,j+\frac{1}{2},k}^l - \sin \theta_{i,j-\frac{1}{2},k} (\rho u)_{i,j-\frac{1}{2},k}^l] + \\ & \frac{\Delta t^2}{\Delta r} \left( \frac{\rho_{i,j,k-\frac{1}{2}}^l u_{i-\frac{1}{2},j,k-\frac{1}{2}}^l u_{i+\frac{1}{2},j,k-\frac{1}{2}}^l}{r_{k-\frac{1}{2}}} - \frac{\rho_{i,j,k+\frac{1}{2}}^l u_{i-\frac{1}{2},j,k+\frac{1}{2}}^l u_{i+\frac{1}{2},j,k+\frac{1}{2}}^l}{r_{k+\frac{1}{2}}} \right) + \\ & \frac{\Delta t^2}{\Delta r} \left( \frac{\rho_{i,j,k-\frac{1}{2}}^l v_{i,j-\frac{1}{2},k-\frac{1}{2}}^l v_{i,j+\frac{1}{2},k-\frac{1}{2}}^l}{r_{k-\frac{1}{2}}} - \frac{\rho_{i,j,k+\frac{1}{2}}^l v_{i,j-\frac{1}{2},k+\frac{1}{2}}^l v_{i,j+\frac{1}{2},k+\frac{1}{2}}^l}{r_{k+\frac{1}{2}}} \right) + \\ & \frac{2\Delta t^2}{\Delta r} w_{i,j,k}^l \left( \frac{\rho_{i,j,k+\frac{1}{2}}^l w_{i,j,k+1}^l}{r_{k+\frac{1}{2}}} - \frac{\rho_{i,j,k-\frac{1}{2}}^l w_{i,j,k-1}^l}{r_{k-\frac{1}{2}}} \right) + \frac{\Delta t^2}{\Delta r} \tan \theta_j \left[ \frac{(\rho v w)_{i,j,k-\frac{1}{2}}^l}{r_{k-\frac{1}{2}}} \right. \\ & \left. - \frac{(\rho v w)_{i,j,k+\frac{1}{2}}^l}{r_{k+\frac{1}{2}}} \right] + \frac{2\Omega_e \cos \theta_j \Delta t^2}{\Delta r} [(\rho u)_{i,j,k-\frac{1}{2}}^l - (\rho u)_{i,j,k+\frac{1}{2}}^l]. \end{aligned} \quad (11)$$

用逐线法求密度方程 (9), 求得  $\rho_{i,j,k}^{l+1}$ , 再由 (8.6) 计算  $P_{i,j,k}^{l+1}$ , 然后由 (8.2)–(8.4) 计算  $(\rho u)_{i+\frac{1}{2},j,k}^{l+1}$ ,  $(\rho v)_{i,j+\frac{1}{2},k}^{l+1}$  和  $(\rho w)_{i,j,k+\frac{1}{2}}^{l+1}$ , 从而由  $\rho_{i,j,k}^{l+1}$  求得  $u_{i+\frac{1}{2},j,k}^{l+1}$ ,  $v_{i,j+\frac{1}{2},k}^{l+1}$  和  $w_{i,j,k+\frac{1}{2}}^{l+1}$ .

取容差为  $\varepsilon$  (本文中取  $\varepsilon = 10^{-4}$ ), 如果

$$\max_{i,j,k} \left( \left| \frac{\rho_{i,j,k}^{l+1} - \rho_{i,j,k}^l}{\rho_{i,j,k}^l} \right| \right) < \varepsilon \quad (12)$$

成立, 则停止迭代, 并取迭代结果为  $U^{n+1}$ , 它同时取作第  $(n+2)$  时间步的迭代初值, 然后进入第  $(n+2)$  时间步. 如果 (12) 式不成立, 则回到 (9) 式继续迭代, 即进入第  $(l+2)$  次迭代步.

## 2.5 时间步长

由于边界采用了显式处理, 时间步长应满足如下的柯朗条件

$$\Delta t < \Delta t_c = \frac{\Delta z}{v_a + w}. \quad (13)$$

FICE 算法属于全隐格式, 时间步长本可以不受限制, 但由于采用近特征边界条件, 时间步长必须满足 (14) 式, 这是 FICE 算法的一个弱点, 但在一定条件下, 这一弱点并不严重, 因为时间步长短有利于提高精度, 而且实现 (10) 式迭代所需要的次数会相应减少, 其计算量不会因时间步长短而增加过多<sup>[2]</sup>. 为此, 在本文的计算中取  $\Delta t = 0.25\Delta t_c$ .

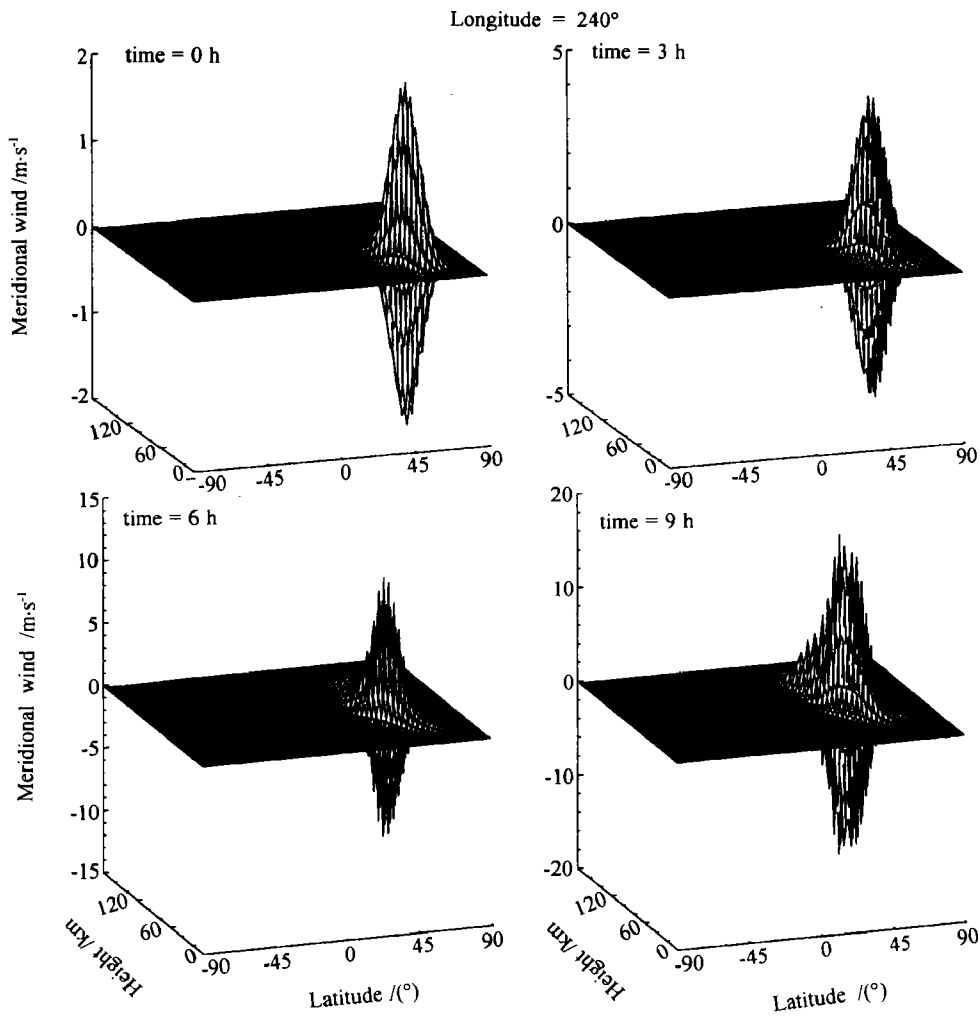


图 1  $\theta$ - $r$  平面上重力波波包的非线性传播

Fig.1 Nonlinear propagation of gravity-wave packet in  $\theta$ - $r$  plane

### 3 模式检验

为了验证该模式的合理性,我们将给定一个向上传播的重力波初始扰动,观察它在三维大气中的传播过程.

设初始背景大气为等温、静止大气,并且满足流体静力学平衡.即

$$T_0 = T_c, \rho_0 = \rho_c e^{-\frac{g}{RT_c} r}, P_0 = R\rho_0 T_0, u_0 = 0 \text{ m/s}, v_0 = 0 \text{ m/s}, w_0 = 0 \text{ m/s}$$

式中,  $T_c = 288.221 \text{ K}$  和  $\rho_c = 1.225 \text{ kg/m}^3$  为常数.

初始时刻,在流体静力学平衡大气中加上一个向上传播的高斯型重力波波包扰动作为初始扰动,其纬向扰动速度具有如下形式:

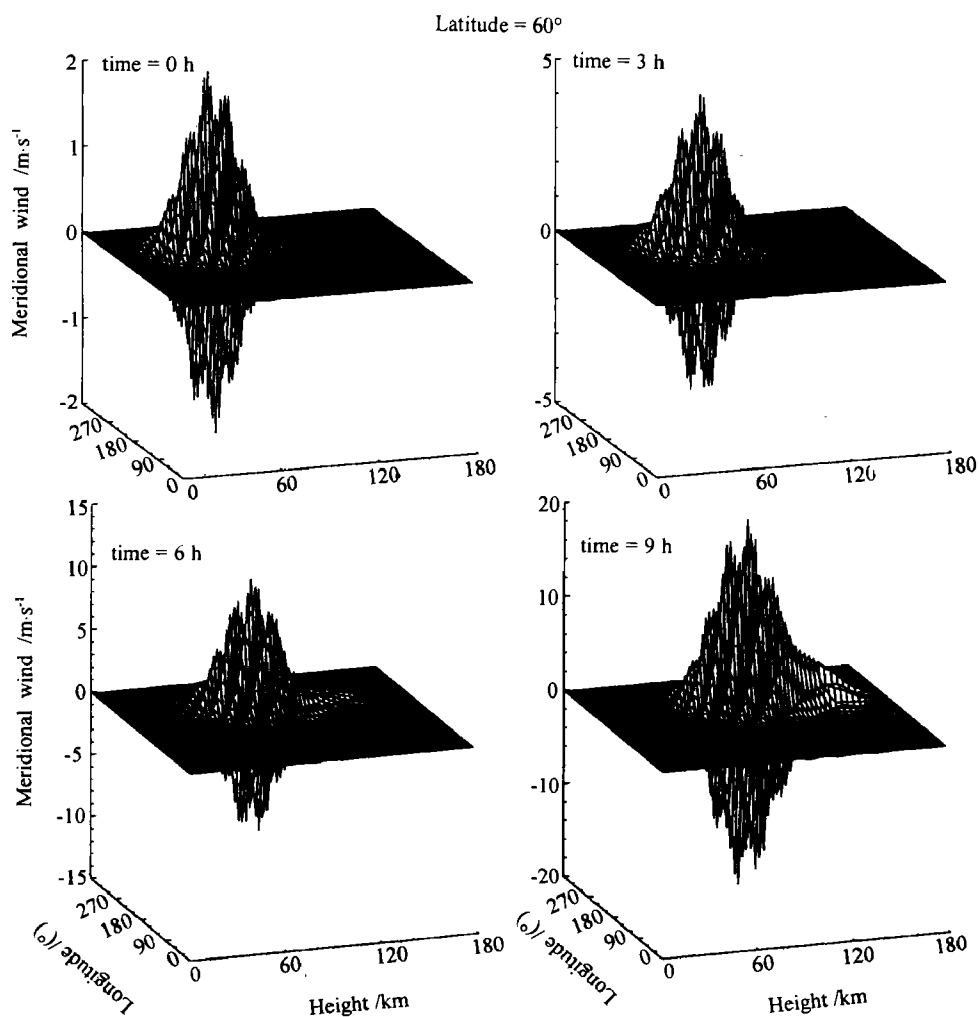


图 2  $\varphi$ - $r$  平面上重力波波包的非线性传播

Fig.2 Nonlinear propagation of gravity-wave packet  $\varphi$ - $r$  in plane

$$u'(r, \varphi, \theta, 0) = u_c e^{-\frac{(r-r_c)^2}{2\sigma_r^2}} e^{-\frac{r_c^2(\theta-\theta_c)^2}{2\sigma_\theta^2}} e^{-\frac{r_c^2 \cos^2 \theta_c (\varphi-\varphi_c)^2}{2\sigma_\varphi^2}} \cdot \cos[k_r(r-r_c) + k_\theta r_c(\theta-\theta_c) + k_\varphi r_c \cos \theta_c(\varphi-\varphi_c)], \quad (14)$$

式中  $u_c$  为初始扰动振幅, 在本文的数值计算中取为 1.5 m/s;  $r_c = r_e + h$ ,  $r_e = 6370$  km 为地球平均半径,  $h$  为初始时刻波包中心位置的高度, 本文中取 60 km;  $\varphi_c$  和  $\theta_c$  为常数, 为了考察本文中边界条件的求解是否合理, 波包中心区在靠近边界的地方, 因此  $\varphi_c$  和  $\theta_c$  分别取为  $240^\circ$  和  $60^\circ$ ;  $k_r$ 、 $k_\varphi$ 、 $k_\theta$  分别为重力波的垂直、经向和纬向波数, 本文中垂直 ( $\lambda_r$ )、经向 ( $\lambda_\varphi$ ) 和纬向 ( $\lambda_\theta$ ) 波长分别取为 15 km、2000 km 和 800 km;  $\sigma_r$ 、 $\sigma_\varphi$  和  $\sigma_\theta$  为波包在三个方向上的半宽度, 本文中取为  $\sigma_r = \lambda_r$ 、 $\sigma_\varphi = \lambda_\varphi$  和  $\sigma_\theta = \lambda_\theta$ . 其他物理量  $v'(r, \varphi, \theta, 0)$ ,  $w'(r, \varphi, \theta, 0)$ ,  $\rho'(r, \varphi, \theta, 0)$ ,  $T'(r, \varphi, \theta, 0)$ ,  $P'(r, \varphi, \theta, 0)$  的初始扰动由重力波的线性偏振关系求出. 将这些物理量的初始扰动和初始背景值加在一起作为初值进行数值计算.

计算结果见图 1 和图 2. 图 1 和图 2 分别为经向扰动风速在  $\theta$ - $r$  和  $\varphi$ - $r$  平面上的非线性传播, 图中重力波波包整体的向上传播清晰可见, 并且波振幅随高明显增长, 这些都与重力波的基本传播特性非常一致. 初始时刻, 波包在 60 km 处, 最大扰动振幅小于 2 m/s, 经过了 9 h 的非线性传播后, 波包传播到大约 100 km, 并且最大扰动振幅增达到大约 20 m/s. 此外, 从计算结果中还可以看到在整个计算过程中, 无论是在内点和边界上, 特别是在纬向边界上, 计算结果都非常稳定, 这表明本文的数值模式在物理上和算法上都是比较令人满意的.

## 4 结 论

将三维球坐标系下的大气基本运动方程作为控制方程, 采用 FICE 格式, 建立起了三维球坐标系下大气运动的全非线性数值模式, 对纬向边界也进行了合理的处理, 并对该模式进行了数值检验. 计算结果显示重力波波包在大气中非线性传播的三维图像, 波包整体向上传播, 波相关扰动速度随高度增加而增长, 并且在整个传播过程中, 内点和边界的值一直很稳定, 这表明文中的三维数值模式是比较成功的.

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# A NUMERICAL SOLUTION METHOD OF NONLINEAR PROPAGATION OF GRAVITY-WAVE PACKET IN A 3-DIMENSIONAL SPHERICAL COORDINATE SYSTEM

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## Abstract

By using FICE scheme and taking the atmospheric basic motion equations as the governing equations, a full-nonlinear numerical model in 3-dimensional spheric coordination is established, and a reasonable polar boundary is specified, the examination of this model is also presented. The calculation result exhibits the essential characteristics of the nonlinear propagation of gravity-wave packet in atmosphere: the whole wave packet propagates upward, the wave-associated disturbance velocity increases with the increasing height.

**Key words**   3-dimension, Global coordination, FICE scheme