

COSMIC PLASMA DYNAMO*

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Abstract

A new dynamo model based on the polarization of plasma is presented in this paper. From the Maxwell equations in a moving medium, a magnetization vector can be caused with Rongon current. The steady solar magnetic field is solved from the equations. On the assumption that the meridional flow is ignored, the distribution of magnetic field is put out. In the model, there is no additional parameter considered. The intensity of magnetic field inside the sun ranges from 1—6 T. The surface magnetic field around the pole is in the order of 1×10^{-3} T, at low latitude the calculated surface magnetic field has the order of 1×10^{-2} T. The maximum magnetic field is around 30° in latitude.

Key words Ionflow, Instability, Relativity

1 Introduction

Magnetic field exists in almost every astronomical objects, from planet to the Sun, stars and galaxies. To solve the problem of generation and maintenance mechanism is an important task in astrophysical study.

Generally, a magnetic field is maintained by currents induced in plasma by its motion across lines of force. therefore, a motion across a magnetic field causes an induced electric field, which drives an electric current by Ohm's law and leads to a magnetic field from Ampere's Law. The magnetic field induces both electric field through Faraday's law and Lorentz's force, which can act the force that drives the motion and so completes the chain of cause and effect.

Many years ago, Cowling^[1] discussed the dynamo problem, by showing that the simplest configuration will not work. Following Cowling's argument, several extensions to the theorem were presented^[2]. the essence of these arguments is that a steady axial symmetric magnetic field cannot be maintained.

Cowling's argument led to fears that no dynamo could possibly work. To be out of the limitation imposed on a workable dynamo by Cowling's theorem, a workable dynamo needs departures from axis-symmetry^[2]. Twenty-five years later, some kinematic dynamo models which have axial asymmetry were established^{[3],[4]}. Since then a swarm of models has appeared. Roberts^[5] has shown that almost all spatially periodic motions can work as dynamo. It should be pointed out, that all these models are not exactly axially symmetry to avoid cowling's argument.

The magnetic field of a spherical body, such as the Sun, may be split into toroidal and poloidal components (B_φ and B_p), and one needs to establish that both components can be generated by a flow. On the Sun the equatorial regions are seen to rotate faster than the polar regions, and such differential rotation tends to pull out a purely poloidal field and

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creates a toroidal flux. The difficulty with an axis-symmetric field is that the poloidal field component cannot be maintained.

A suggestion for generating B_p from B_φ was put forward in a fundamental paper by Parker^[6]. He pointed out that as rising bulbs of plasma expand they also tend to rotate because of the Coriolis force. If the bulbs carry flux tubes, the twist converts toroidal fields into poloidal ones. The rate of generation of B_p is proportional to B_φ , and Parker modelled the net effect of many convection cells by adding an electric field, $E_\varphi = \alpha B_\varphi$. Then the dynamo action is possible. It is so called α -effect. The mathematical formalism of mean field theory was applied to the Sun by Steenbeck & Krause^[7], who showed that this effect leads to the appearance, in the induction equation for the mean field of an additional source term, which they wrote as $\nabla \times (\alpha B)$ ^{[8],[9]}. Numerous mean field models were subsequently produced for the α -effect^[10].

Following the advent of helioseismology, the solar internal differential rotation is no longer a free ingredient of dynamo models. The helioseismic inversions of the solar internal rotation^{[11]–[13]} have shown that the strongest shear is associated with the so-called tachocline, i.e., the rotation shear layer located immediately beneath the core-envelope interface. Traditionally, it has been argued that the α -effect will be strongly reduced once the energy of the magnetic field becomes comparable with that of the small-scale fluid motions giving rise to the α -effect. For conditions typical of deep solar convection, this equipartition field strength is at most $B_{eq} = 1$ T. In the framework of mean field dynamo modelling, this has been taken to imply that once the large-scale fields reaches B_{eq} , the dynamo saturates^[14]. From there, one may argue that the process leading to the formation of a rope of concentrated toroidal flux from a diffused background field can presumably account for the further intensification by the models of rising flux ropes. However, numerical experiments have suggested that the α -effect is quenched long before reaching equipartition with the large-scale mean magnetic field^[15]. Then a class of mean field models known as interface dynamos^[16] has been shown to avoid this difficulty^[17], but the full nonlinear behavior of these models remains to be explored.

An alternate regeneration mechanisms is based on flux rope. The twist imparted by the Coriolis force on the rising flux ropes drives a mean zonal electric current, in turn inducing a mean poloidal field. This is analogous to the α -effect of mean field theory, except that the process operates on toroidal flux ropes at much large spatial scales. A model of the solar cycle relying essentially on this mechanism was first proposed by Babcock^[18] and later elaborated on by many authors^{[19]–[22]}. Dynamo models relying on this poloidal field regeneration mechanism are labelled as “Babcock-Leighton dynamos”. The essential ingredient of such a model is the transport of the poloidal flux newly generated in the surface layers down to the region of strong shear in order to generate the toroidal field. It is a promising model in explain some features of the solar magnetic field. However, the flux rope need more study, even in observation.

Besides the mean magnetic field, there is no reasonable dynamo theory for the small scale magnetic field on the Sun. The weak turbulence in galaxies leave a puzzle for the generation of galactic magnetic field. For such a general existence of magnetic field in the universe, we have reason to find a general theory to explain it.

Classic physics pointed out that there are two possibility for the generation of magnetic field: permanent magnetic object and current. For astronomical objects, current is the only possibility. There are several kinds of currents from different physical mechanism. Generally, the current is considered as the motion of charged particles in electrical field, this

is the conduction current. Another kind of current is the polarization current, this is from the polarization of molecules in electrical field. Obviously, molecules in stars with too high temperature is impossible. For an axis-symmetric rotation body, conduction current cannot maintain magnetic field^[1]. Therefore, the magnetic field in astronomical objects cannot be generated from conduction current. However, for the plasma, polarization current which due to the drift motion of charged particles in magnetic field may be possible. The effect of drift motion of charged particles in magnetic field is considered as the ‘effective dielectricity’ of plasma.

We will begin from the drift motion, which is considered as the ‘polarization of plasma’, to discuss a kind of dynamo model in the present paper. In Section 2 and 3, we present the basic physics discussion. The full equations are in Section 4. Then we use our result to model the internal solar magnetic field in Section 5.

2 Polarization of Plasma

Plasma is an assemblage of charged particles. We consider the equation of motion of charged particles in a magnetic field. In ordinary coordinates (x, y, z) , the direction of magnetic field is assumed in z -direction. There is a slowly varied uniform electrical field $\mathbf{E}$ in x -direction:

$$\mathbf{E} = iE_0 \cos \omega t. \quad (1)$$

The equation of motion of charged particle is:

$$m \frac{d\mathbf{v}}{dt} = q[\mathbf{E}(y) + \mathbf{V} \times \mathbf{B}]. \quad (2)$$

Then the equation of motion transverse to the magnetic field are:

$$\dot{v}_x = \frac{qB}{m} v_y + \frac{q}{m} E_x(y), \quad (3)$$

$$\dot{v}_y = -\frac{qB}{m} v_x. \quad (4)$$

That is:

$$\ddot{v}_x = -\omega_c^2 v_x \pm \frac{\omega_c}{B} E_x(y), \quad (5)$$

$$\ddot{v}_y = -\omega_c^2 v_y - \frac{\omega_c}{B} E_x(y), \quad (6)$$

where

$$\omega_c = \frac{qB}{m} \quad (7)$$

is the Larmor frequency of particle and $E_x(y)$ is the electrical field where particle is at.

We define:

$$\mathbf{v}_{dp} = \pm \frac{1}{\omega_c B} \dot{E}_x, \quad (8)$$

The sign $\pm$ depends on the charge of particles,

$$\mathbf{v}_{de} = -\frac{E_x}{B}. \quad (9)$$

Since E_x varies with time, the motion of particles is different at different time. We can write the equations as:

$$\ddot{v}_x = -\omega_c^2 (v_x - v_{dp}), \quad (10)$$

$$\ddot{v}_y = -\omega_c^2(v_y - v_{de}). \quad (11)$$

The solutions of above equations are the sum of circular and drift motion. When the varying frequency of electrical field is much smaller than the Larmor frequency of particles, we can get the simple expression for the solution:

$$v_x = v_\perp \cos(\omega_c + \alpha) + v_{dp}, \quad (12)$$

$$v_y = -\pm v_\perp \sin(\omega_c + \alpha) + v_{de}. \quad (13)$$

The guiding center of the particles has the common drift $\mathbf{E} \times \mathbf{B}$, which is independent of the charges of particles. However, there is another drift along the direction of the variation of electrical field.

$$v_{dp} = \pm \frac{1}{\omega_c B} \frac{d\mathbf{E}}{dt}. \quad (14)$$

For particles with different charge, v_{dp} is in different direction. The variation of electrical field makes the particles with positive and negative charge move in opposite direction. The opposite motions of charged particles cause the polarization current of plasma, or called the drift current $\mathbf{j}_p$.

$$\mathbf{j}_p = n_i q_i v_{dp,i} + n_e q_e v_{de,e} = \frac{\rho}{B^2} \frac{d\mathbf{E}}{dt}, \quad (15)$$

where ρ is the density of plasma. Generally v_{dp} is named polarization drift.

The polarization current $\mathbf{j}_p$ can be considered as part of total current $\mathbf{J}$ in Maxwell equations. If $\mathbf{J}$ represent the total current, $\mathbf{j}$ is the conduction current, then we have:

$$\mathbf{J} = \mathbf{j}_p + \mathbf{j}. \quad (16)$$

Considering $\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}$, we can write:

$$\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}_p + \mathbf{j}. \quad (17)$$

Substituting the expression of $\mathbf{j}_p$ into the above equation, we can get:

$$\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} = \varepsilon_0 \left(1 + \frac{\rho}{\varepsilon_0 B^2} \right) \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}. \quad (18)$$

Since

$$\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}, \quad (19)$$

where $\mathbf{D}$ is the electrical displacement vector. According to $\mathbf{D} = \varepsilon \mathbf{E}$, we can have the effective electric capacity, generally named dielectric constant of plasma:

$$\varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon_0 \left(1 + \frac{\rho}{\varepsilon_0 B^2} \right). \quad (20)$$

After we consider the polarization drift, plasma can be thought as common dielectric medium with the dielectric constant as equation (20).

Equation (20) is the expression of dielectric constant of plasma transverse the magnetic field. In general, magnetic field is not in the direction of z -direction. If the magnetic field is expressed as (B_x, B_y, B_z) , the di-electricity is not uniform in different directions. We can follow the equation (20) to get the dielectric constant in different direction. The dielectric

constant should be a tensor:

$$\varepsilon = \varepsilon_0 \begin{pmatrix} 1 + \frac{\rho}{\varepsilon_0(B_y^2 + B_z^2)} & 1 & 1 \\ 1 & 1 + \frac{\rho}{\varepsilon_0(B_x^2 + B_z^2)} & 1 \\ 1 & 1 & 1 + \frac{\rho}{\varepsilon_0(B_x^2 + B_y^2)} \end{pmatrix}. \quad (21)$$

3 The Electrodynamics of Dielectric Medium

In the discussion of dynamo model, plasma is considered as fluid. The basic theory is magnetohydrodynamics. Maxwell equations can describe the macro-scale electric field and magnetic field of plasma. However, when we use the Maxwell equations in plasma, the polarization effect of plasma is generally ignored, the relative dielectric constant of plasma is unit. However, according to the above discussion, plasma has large polarization in magnetic field, the Maxwell equations in moving medium must be applied.

Generally, the basic Maxwell equations are described as:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (22)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}, \quad (23)$$

$$\nabla \cdot \mathbf{D} = \rho_e, \quad (24)$$

$$\nabla \cdot \mathbf{B} = 0. \quad (25)$$

where $\mathbf{E}$ is the strength of electric field, $\mathbf{D}$ is electric displacement vector, $\mathbf{B}$ is the strength of magnetic induction, $\mathbf{H}$ is the magnetized force, ρ_e is the real macro-scale charge density, $\mathbf{j}$ is the real macro-scale conduction current density. To complete the equations, we should have the three relations for $\mathbf{D}$, $\mathbf{E}$, $\mathbf{B}$, $\mathbf{H}$, and $\mathbf{j}$. These three relations depend on the property of medium. In static coordinate system, these equations are expressed as:

$$\mathbf{B} = \mu \mathbf{H}, \quad (26)$$

$$\mathbf{D} = \varepsilon \mathbf{E}, \quad (27)$$

$$\mathbf{j} = \sigma \mathbf{E}. \quad (28)$$

where μ is magnetic diffusivity, ε is dielectric constant, σ is conductivity.

However, these equations depends on the coordinate system. If we describe the Maxwell equations with $\mathbf{E}$ and $\mathbf{B}$, the form of the the Maxwell equations will depends on the coordinate systems with different velocities.

From the relativistic transformation (Rosser, 1971), we know the following relations:

$$\mathbf{E}'_{//} = (\mathbf{E} + \mathbf{v} \times \mathbf{B})_{//}, \quad (29)$$

$$\mathbf{E}'_{\perp} = \gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B})_{\perp}, \quad (30)$$

$$\mathbf{D}'_{//} = \left(\mathbf{D} + \frac{1}{c^2} \mathbf{v} \times \mathbf{H} \right)_{//}, \quad (31)$$

$$\mathbf{D}'_{\perp} = \gamma \left(\mathbf{D} + \frac{1}{c^2} \mathbf{v} \times \mathbf{H} \right)_{\perp}. \quad (32)$$

The variants with “'” represent the quantity in a static coordinate system Σ' . The variants without “'” are the ones in a coordinate system Σ moving with velocity $\mathbf{v}$ relative

to Σ' .

We have

$$\left(D + \frac{1}{c^2} \mathbf{v} \times \mathbf{H}\right) = \varepsilon(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (33)$$

$$\left(\mathbf{B} + \frac{1}{c^2} \mathbf{v} \times \mathbf{E}\right) = \mu(\mathbf{H} - \mathbf{v} \times \mathbf{D}), \quad (34)$$

$$\gamma(\mathbf{J} - \mathbf{j}_p)_{//} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})_{//}, \quad (35)$$

$$(\mathbf{J} - \mathbf{j}_p)_{\perp} = \sigma\gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B})_{\perp}, \quad (36)$$

where “ $\perp$ ” represents the quantity perpendicular to $\mathbf{v}$ and “ $//$ ” represents parallel to $\mathbf{v}$. $\mathbf{J}$ is total current density, $\mathbf{j}_p$ is displacement current density, $\mathbf{j} = \mathbf{J} - \mathbf{j}_p$ is conduction current density. Equations (33) (34) (35) and (36) are the equations in Σ .

Considering the fact that $\mathbf{H}$ and $\mathbf{B}$ are only the first order of v/c , we can ignore the terms with higher order of v/c . Then we can get the equations in non-relativistic case:

$$\mathbf{D} = \varepsilon \mathbf{E}, \quad (37)$$

$$\mathbf{B} = \mu \mathbf{H} - (\varepsilon\mu - 1)(\mathbf{v} \times \mathbf{E})/c, \quad (38)$$

$$(\mathbf{J} - \mathbf{j}_p) = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \quad (39)$$

We notice that in non-magnetized medium, μ is the value in vacuum. if the dielectric constant is near the value in vacuum, that is $\varepsilon = 1$, the last term of equation (38) can be ignored, we can get the common relation between $\mathbf{B}$ and $\mathbf{H}$.

3.1 The Polarization Vector and Magnetization Vector of Plasma

Generally, the polarization vector $\mathbf{P}$ is defined as polarization of electric medium in external electrical field. The polarization is due to the rearrangement of polarized molecules in external electric field and the departure of positive and negative charges in atoms and molecules.

In plasma, we cannot understand the polarization as electrical dipole. However, when there is a varied external electrical field, the ions and electrons moves in different direction, this causes the polarized current, we can describe the polarized current with the variation of $\mathbf{D}$. Therefore, we can define the polarization of plasma as:

$$\mathbf{P} = \mathbf{D} - \varepsilon \mathbf{E}. \quad (40)$$

With above equation, we can transform $\mathbf{P}$ in different coordinate systems. $\mathbf{P}'$ is the polarization vector in a static coordinate system relative to medium, $\mathbf{P}$ is the polarization vector in a coordinate system moving with velocity $\mathbf{v}$ relative to medium.

In the coordinate system Σ' ,

$$\mathbf{P}' = \mathbf{D}' - \varepsilon_0 \mathbf{E}'. \quad (41)$$

Using the transformation of $\mathbf{D}$, $\mathbf{E}$ in equations (29)—(32), we can get the equation about $\mathbf{P}$ from Σ' to Σ .

If $\mathbf{P}_{//}$ and $\mathbf{P}_{\perp}$ represent the polarization parallel to $\mathbf{v}$ and perpendicular to $\mathbf{v}$ respectively, we can get:

$$\mathbf{P}_{//} = \mathbf{P}'_{//}, \quad (42)$$

$$\mathbf{P}_{\perp} = \gamma\left(\mathbf{P}'_{\perp} + \frac{\mathbf{v} \times \mathbf{M}'_{\perp}}{c^2}\right). \quad (43)$$

For plasma, it is a non-magnetized medium, magnetization vector $\mathbf{M}' = 0$. In the

non-relativistic case, $\gamma = 1$, we can write:

$$\mathbf{P} = \mathbf{P}'. \quad (44)$$

Defining the magnetization vector $\mathbf{M}$ as:

$$\mathbf{M} = \frac{\mathbf{B}}{\mu_0} - \mathbf{H}. \quad (45)$$

In Σ' ,

$$\mathbf{M}' = \frac{\mathbf{B}'}{\mu_0} - \mathbf{H}'. \quad (46)$$

we can get

$$\mathbf{M}_{//} = \mathbf{M}'_{//}, \quad (47)$$

$$\mathbf{M}_{\perp} = \gamma(\mathbf{M}'_{\perp} - \mathbf{v} \times \mathbf{P}'_{\perp}). \quad (48)$$

When $\gamma = 1$, equation (48) can be written as:

$$\mathbf{M}_{\perp} = \mathbf{M}'_{\perp} - \mathbf{v} \times \mathbf{P}'_{\perp}. \quad (49)$$

The term $\mathbf{v} \times \mathbf{P}'$ is called Rontgen current for polarized medium. It contributes magnetism to the medium. The above description is referred to Rossel^[23].

3.2 The Electrodynamics in Non-relativistic Case

We consider plasma as medium with dielectric tensor ε . The terms with v^2/c^2 are ignored, and $\gamma = 1$ is assumed everywhere in non-relativistic case. The Maxwell equations in Σ are:

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}, \quad (50)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (51)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (52)$$

$$\nabla \cdot \mathbf{D} = 0. \quad (53)$$

In Σ' , $\mathbf{M}' = 0$, then

$$\mathbf{B}' = \mu_0 \mathbf{H}', \quad (54)$$

and from equations (47) and (49), the following relations are obtained.

$$\mathbf{M}_{//} = \mathbf{M}'_{//} = 0, \quad (55)$$

$$\mathbf{M} = -\mathbf{v} \times \mathbf{P}'_{\perp}. \quad (56)$$

From equation, we have:

$$\mathbf{P} = \mathbf{P}', \quad (57)$$

then

$$\mathbf{M} = \mathbf{M}_{\perp} = -\mathbf{v} \times \mathbf{P}'_{\perp} = \mathbf{P} \times \mathbf{v}. \quad (58)$$

In Σ , we have

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} = \frac{\mathbf{B}}{\mu_0} - \mathbf{P} \times \mathbf{v}. \quad (59)$$

Substitute it into equation (50), we have

$$\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}, \quad (60)$$

or

$$\nabla \times \mathbf{B} = \mu_0 \left[\mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{P}}{\partial t} + \nabla \times (\mathbf{P} \times \mathbf{v}) \right]. \quad (61)$$

Equations (61), (51), (52) and (53) are the equations for moving medium in non-relativistic case^[23].

In Σ' , we have

$$\mathbf{P}' = \mathbf{D}' - \varepsilon_0 \mathbf{E}' = \varepsilon'_r \varepsilon_0 - \varepsilon_0 \mathbf{E}' = \varepsilon_0 (\varepsilon'_r - \mathbf{I}) \mathbf{E}', \quad (62)$$

where ε'_r is the dielectric tensor of static plasma. Since the dielectric tensor of plasma depends only on the density of plasma and magnetic field, we can get:

$$\varepsilon_r = \varepsilon'_r = \begin{pmatrix} 1 + \frac{\rho}{\varepsilon_0(B_y^2 + B_z^2)} & 1 & 1 \\ 1 & 1 + \frac{\rho}{\varepsilon_0(B_x^2 + B_z^2)} & 1 \\ 1 & 1 & 1 + \frac{\rho}{\varepsilon_0(B_x^2 + B_y^2)} \end{pmatrix}. \quad (63)$$

Because $\mathbf{P} = \mathbf{P}'$, we can get

$$\mathbf{P} = \varepsilon_0 (\varepsilon_r - \mathbf{I}) (\mathbf{E} + \mathbf{v} \times \mathbf{B}). \quad (64)$$

From equation (61) and (63), we can get

$$\nabla \times \mathbf{B} = \mu_0 \left\{ \mathbf{j} + \varepsilon_0 \varepsilon_r \frac{\partial \mathbf{E}}{\partial t} + \varepsilon_0 (\varepsilon_r - \mathbf{I}) \frac{\partial (\mathbf{E} + \mathbf{v} \times \mathbf{B})}{\partial t} + \nabla \times \{ \varepsilon_0 (\varepsilon_r - \mathbf{I}) [(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \times \mathbf{v}] \} \right\}. \quad (65)$$

In equation (65), we can find that an additional magnetization vector, which comes from the motion of the plasma, can contribute magnetic field to the plasma. With the conduction current density

$$\mathbf{j} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (66)$$

and

$$\nabla \times \mathbf{B} = 0, \quad (67)$$

we can find the magnetic dynamo from the moving plasma.

4 The Magnetic Dynamo for the Sun

We suppose the Sun is a plasma sphere. To get the steady magnetic field, we assume $\partial/\partial t = 0$. If there is no external electrical field in the Sun, then we can get $\mathbf{E} = 0$. Equation (65) can be written as:

$$\nabla \times \mathbf{B} = \mu_0 \{ \mathbf{j} + \nabla \times \{ \varepsilon_0 (\varepsilon_r - \mathbf{I}) [(\mathbf{v} \times \mathbf{B}) \times \mathbf{v}] \} \}. \quad (68)$$

We suppose the conductivity of plasma is infinite, so the conduction current is assumed as $\mathbf{j} = 0$ in a steady case.

$$\nabla \times \mathbf{B} = \mu_0 \nabla \times \{ \varepsilon_0 (\varepsilon_r - \mathbf{I}) [(\mathbf{v} \times \mathbf{B}) \times \mathbf{v}] \}. \quad (69)$$

Under the assumption of axis-symmetry, using spherical polar coordinates (r, θ, φ) , and working in the kinematic regime ($\mathbf{v}$ is given), the magnetic field and flow fields can be written as

$$\mathbf{B} = B_\varphi \mathbf{e}_\varphi + B_r \mathbf{e}_r + B_\theta \mathbf{e}_\theta, \quad (70)$$

$$\mathbf{v} = v(r, \theta) + r \sin \theta \Omega(r, \theta) \mathbf{e}_\varphi, \quad (71)$$

where B_φ , B_r and B_θ correspond to the components of the magnetic field and the meridional circulation $\mathbf{v}(r, \theta)$ and differential rotation $\Omega(r, \theta)$ to the poloidal and toroidal parts of the total flow field $\mathbf{v}$.

The simplest solution of equation (69) is that:

$$\mathbf{B} = \mu_0 \varepsilon_0 (\varepsilon_r - \mathbf{I}) [(\mathbf{v} \times \mathbf{B}) \times \mathbf{v}]. \quad (72)$$

With equation (69), and we suppose the meridional flow is ignored, the solutions of steady magnetic field of the Sun can be obtained as:

$$B_\varphi = 0, \quad (73)$$

$$B_r = \mu_0 \rho v_\varphi^2 B_r / B_\theta^2, \quad (74)$$

$$B_\theta = \mu_0 \rho v_\varphi^2 B_\theta / B_r^2. \quad (75)$$

The solution is

$$B_r^2 = B_\theta^2 = \mu_0 \rho v_\varphi^2. \quad (76)$$

We assign the angle from the rotation axis is θ , the rotation velocity can be expressed as:

$$v_\varphi = r \sin(\theta) \Omega, \quad (77)$$

$$B_\varphi = 0. \quad (78)$$

In the kinematic regime, the components of the large-scale velocity fields are prescribed. We use the following expression for the differential rotation Ω :

$$\Omega = \Omega_c + \frac{1}{2} \left[1 + \operatorname{erf} \left(2 \frac{r - r_c}{d_1} \right) \right] (\Omega_s - \Omega_c), \quad (79)$$

where $\Omega_s = \Omega_{\text{eq}} + a_2 \cos^2 \theta + a_4 \cos^4 \theta$ is the surface latitudinal differential rotation and $\operatorname{erf}(x)$ is the error function. The other parametric values are set as $\Omega_c/2\pi = 432.8$, $\Omega_{\text{eq}}/2\pi = 460.7$, $a_2 = -62.69$, and $a_4 = -67.13$ nHz and $r_c = 0.7 R_s$ and $d_1 = 0.05 R_s$ ^[14], which closely resembles the best-fit helioseismic solution of Charbonneau et al^[24]. This defines a solar-like differential rotation profile, characterized in the “envelope” by purely latitudinal differential rotation (Ω_s) with equatorial acceleration, smoothly matching across a “tachocline” of thickness d_1 on a “core” rotating rigidly at a rate Ω_c . The parameter r_c corresponds to the central radius of the tachocline, where the radial shear is confined. This differential rotation profile resembles rather closely that inferred by fully two-dimensional helioseismic inversions^{[13],[24]}

With the density profile of the convection zone by $\rho(r) = \rho_c [(R_s/r) - 1]^m$, m is used as 0.5, we assume the relation hold till the surface. Choosing $\rho_c = 1.5 \times 10^{-1} \text{ kg/m}^{-3}$ at 0.7 solar radius, we can get the configuration of magnetic field inside the Sun. Each line represents the magnetic field depending on the radius in different angle θ from the polar. The magnetic field decreases towards the solar surface. It increases toward the equator. Figure 1 is the magnetic field distribution in different radius inside the Sun. The strength of magnetic field decrease toward the surface of the Sun. At angle 60° near the core, the strength of the magnetic field is about $1.2 \times 10^{-2} \text{ T}$; The magnetic field is about 0.1 T near the surface with the above assumption of solar density. Eventually, the density at the solar photosphere is much lower than the result from the density formula. If we use the photosphere density as $\rho = 10^{-8} \text{ g}\cdot\text{cm}^{-3}$, the average surface toroidal magnetic field is about $1 \times 10^{-3} \text{ T}$. Fig.2 is the surface toroidal magnetic field of the Sun. The angle θ is from the polar of the Sun. It shows that the magnetic field is stronger near the equator of the Sun.

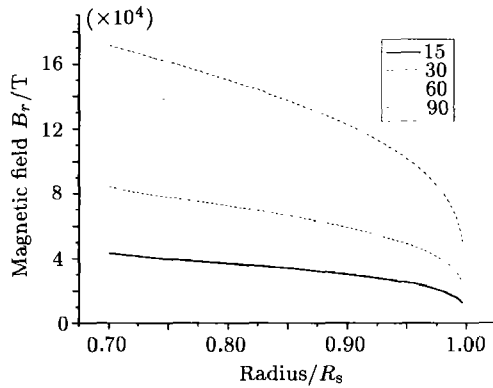


Fig.1 The distribution of the components of $B_r = B_\theta$ inside the Sun. B_r is in the unit of 10^{-4} T and the radius is relative to the solar radius. The linestyles represent different latitude

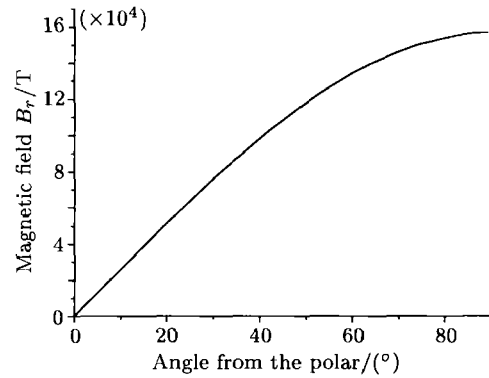


Fig.2 The solar surface magnetic field varies with different angle from the polar. Magnetic field in the unit of 10^{-4} T and the angle in the unit of degree

5 Discussion

(1) The polarization of plasma is the main reason for the generation and maintenance of magnetic field of astronomical objects. In the present paper, we start from the polarization of plasma to model the magnetic dynamo of the Sun. Physically, the polarization of plasma is due to the plasma particles undergo a finite mean displacement, in the direction of the electric field. This displacement is opposite for particles of opposite charge. The precise shape of each particle trajectory depends on whether the electric field rises slowly or rapidly compared with the cyclotron period. The opposite displacement of positive and negative plasma particles is the polarization^{[25],[26]}. Though the polarization of plasma is different from an ordinary dielectric medium, we can consider plasma as an ordinary dielectric medium if we set the effective dielectric tensor for plasma.

(2) From our dynamo model, we can calculate the steady magnetic field in its simplest form. When we ignore the meridional flow of the Sun, the azimuthal magnetic field is zero. In this case, the components of magnetic field B_r and B_θ have the same value. The mean field within about 35° of each pole has an intensity of the order 1×10^{-3} T at the surface. The maximum intensity of surface magnetic field is near the equator of the sun.

(3) The distribution of magnetic field in different radius shows that the internal magnetic field is stronger than that at surface. The average intensity of magnetic field inside the Sun is about 1 T to 8 T. The magnetic field is in the direction of meridian. There is no azimuthal component in the assumption that the meridional flow is ignored. We can use the result to explain the Ω -loops emerging through the photosphere. If the magnetic field forms flux ropes inside the Sun, the participating toroidal flux ropes should have strengths in the range 6–10 T^{[27]–[30]}. Our model can meet the need of toroidal magnetic field for emerging of Ω -loops. However, in our model, the azimuthal field B_ϕ is ignored, the tilts, though the Coriolis force can provide it, may not be incompatible with Joy's Law. We hope this incompatibility can be solved when we consider the meridional flow inside the Sun and provide the azimuthal component.

Though we hope our dynamo model can explain the magnetic generation and maintenance in the Sun, the first result of our study cannot explain every detail of observed solar

magnetic field. We expect the further study can present more detailed result.

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