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An Improved Double r-iteration IOD Method for GEO UCTs Based on SBSS System

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Abstract The purpose of initial orbit determination, especially in the case of angles-only data for observation, is to obtain an initial estimate that is close enough to the true orbit to enable subsequent precision orbit determination processing to be successful. However, the classical angles-only initial orbit determination methods cannot deal with the observation data whose Earth-central angle is larger than 360°. In this paper, an improved double r-iteration initial orbit determination method to deal with the above case is presented to monitor geosynchronous Earth orbit objects for a space-based surveillance system. Simulation results indicate that the improved double r-iteration method is feasible, and the accuracy of the obtained initial orbit meets the requirements of re-acquiring the object.

Key words Double r-iteration, Initial orbital determination, Geosynchronous object, Uncorrelated targets, Space-based space surveillance

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0 Introduction

The first satellite of the USA's Space-based Space Surveillance (SBSS) system, launched on September 26, 2010, carried the newest Space-based Visible (SBV) sensor designed for conducting space surveillance from a space platform. The satellite improves the ability to detect objects in space, especially Geosynchronous Earth Orbit (GEO) objects. Using SBV sensor which provides high-accuracy angle measurements (right ascension and declination), SBSS system can detect newly launched satellites, search for lost or unknown space objects, detect and track debris, catalog maintenance for space objects, and provide all-weather, day and night, nearly real-time space situational awareness data[1−2].

The SBV sensor has two properties that make it uniquely suitable for GEO object surveillance. First, the SBV sensor is on a space platform and can access to the entire geosynchronous belt. Second, the wide field of view of the sensor allows efficient search operations and data to be collected on multiple Resident Space Objects (RSOs) simultaneously. Surveillance data of an SBV sensor are collected in a sidereal track mode, in which the stars appear as point sources and the RSOs appear as streaks. A streak is called an Uncorrelated Target (UCT) if it does not correspond to the predicted position of any known object to within a threshold of 125 millidegrees[3−4].

To recovery a UCT, its initial orbit must be

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determined firstly. There are three classical angles-only Initial Orbital Determination (IOD) methods: Laplace's, Gauss's and double r-iteration. These methods use three points of chronologically ordered angle measurements to solve for the six Cartesian position and velocity components at the epoch of the second point. Laplace's method fits only the middle point, it works poorly for near-Earth satellites. Gauss's method fit data to all three points and is better suited for near-Earth satellites, but the central angle (from the Earth's center) subtended by the three points should be less than 60°. The double r-iteration is effective for large spreads in the data, but the central angle should be less than 360°[5−6].

A single UCT cannot completely describe the kinetic or orbital characteristics of an object and cannot determine its initial orbit. Thus multiple UCTs belonging to the same object must be used on IOD[7]. However, for a GEO object with large inclination or low eccentricity, an SBV sensor can collect one or none observation arc per day. It introduces another problem that the central angle subtended by the three points is more than 360°, and none of the angles-only IOD methods can be used. To overcome the disadvantage, this paper presented an improved double r-iteration method.

1 Problem Statement

The efficiency of a space-based space surveillance system can be improved significantly if a search fence is established in the clustering region (called pinch points in Ref.[3]) for the crowding of Right Ascensions of the Ascending Nodes (RAAN) for GEO objects. These pinch points will become clearly visible, if the population of the GEO belt is viewed over a 24-hour period, as shown in Figure 1. In this figure, each object progresses from left to right along a sinusoidal trajectory. The contours indicate the number of distinct satellite passing through a 2° × 2° region of inertial space over a 24-hour period, which is the field of view of the SBV sensor[8]. The pinch points are centered at 0° in declination and approximately 60° and −120° in right ascension. The dataset used for Figure 1 contains 413 GEO objects in orbit at the epoch of December 10, 2009.

The observation frequency for all GEO objects is an indicator for the efficiency of an SBSS system. Figure 2 displays two pie charts for the observation frequency. There are 413 GEO satellites observed by an SBV sensor whose orbital period is 97 min, and the wide of the search fence is 60°. The field of view of the SBV sensor in chart (a) is 2° × 2°, and in chart (b) is 4° × 4°.

In Figure 2, if the field of view of the SBV sensor is 2° × 2° and 4° × 4°, about two-thirds and three quarters of GEO objects can be observed for more than two times per day, respectively. When the obse-

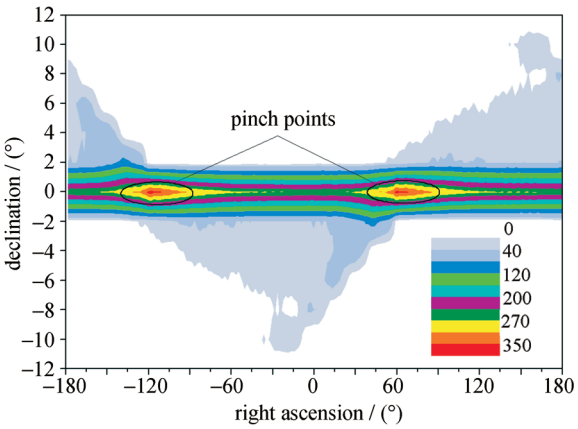


Fig. 1 Geosynchronous pinch points

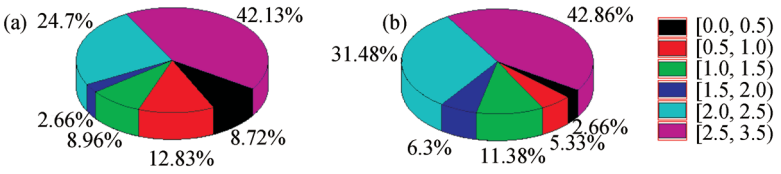


Fig. 2 Pie chart for observation frequency for all GEO objects

rvation frequency is more than 2, we can use Gauss's IOD method to determine its initial orbit[9]. The three points of angle measurements involve the first observation data of Arc1, and the first and last observation data of Arc2, as shown in Figure 3(a). In this figure, $\boldsymbol{R}_i$ is the geocentric position vector of the SBV sensor, $\boldsymbol{\rho}_i$ is the vector in the observation direction, and $\boldsymbol{r}_i$ is the geocentric position vector of the observed satellite, where $i = 1, 2, 3$. Arc1 and Arc2 are observed in the same day, and the central angle θ_{13} subtended by the three points is less than 60°, which is the wide of search fence.

However, there are also about one-third and a quarter of GEO objects observed less than two times per day, when the field of view of the SBV sensor is $2° \times 2°$ and $4° \times 4°$, respectively. This GEO object with little observation frequency has a large inclination or low eccentricity. We cannot collect two arcs in the same day, and single observation arc cannot determine its initial orbit. So arcs from separate days must be used, as shown in Figure 3(b). In this figure, Arc1, Arc2 and Arc3 are collected in different days, and three points of angle measurements are selected from three arcs respectively. θ_{12} and θ_{23} which are the central angles between two adjacent points is more or less than 360°, their sum is much more than 360°, thus the classical Gauss's and double r-iteration IOD method both failed.

2 Improved Double r-iteration Method

To solve the IOD problem whose central angle subtended by three points is much more than 360°, we presented an improved double r-iteration method. The classic double r-iteration method does not take into account of the arcs from separate days when calculates the angles between the three points. It's the reason why classic double r-iteration method is failed. In order to take into account of the case of separate days, we introduce a specific number of complete

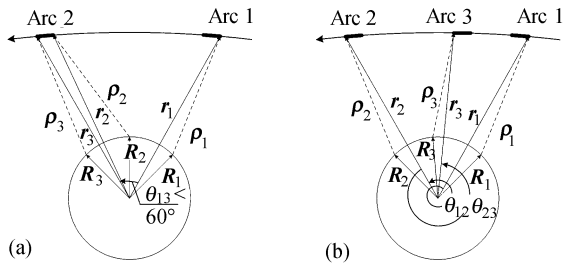


Fig. 3 Position vector geometry. (a) Arc1 and Arc2 are observed in the same day, (b) Arc1, Arc2 and Arc3 are observed in different days

revolutions for the three points into the double r-iteration method.

IOD methods are dependent on the geometry and noise of the observation data[10]. Thus the three points for IOD must come from three different arcs, and their Geometric Dilution of Precision (GDOP) must be maximal, that is,

$$G = \boldsymbol{\rho}_1 \cdot (\boldsymbol{\rho}_2 \times \boldsymbol{\rho}_3), \tag{1}$$

where G is GDOP, and it must be maximal[11].

The process of the classic double r-iteration method (which is the basic of this paper) can refer to the Algorithm 52 in Ref. [5]. It is comprised of preliminary orbit search, computation of orbital parameters, determination of the accuracy of the computed orbit, and differential correction[6]. To solve the IOD problem, we improved the first and second steps shown as follows.

(1) In the first step, the number of complete revolutions for the three points is calculated, that is,

$$N_{jk} = (t_j - t_k)/86164.1, \tag{2}$$

where N is the number of complete revolutions, t is the time of the observation data, 86164.1 is the orbital period of a GEO object in unit of seconds, $j = 2, 3$ and $k = 1, 2$.

(2) During the preliminary orbit search, initial estimates of the observed object position vector magnitudes at the times of the first two points, $|\boldsymbol{r}_1|$ and $|\boldsymbol{r}_2|$, are determined. For a GEO object, the orbit

altitude is determined approximately, thus we can set the initial estimates as

$$\begin{cases} |\boldsymbol{r}_1| = 6.6\,R_{\mathrm{E}}, \\ |\boldsymbol{r}_2| = 6.61\,R_{\mathrm{E}}, \end{cases} \tag{3}$$

where R_{E} is the Earth radius.

(3) In the second step, three formulae are improved. The first formula is the calculation of the true anomaly differences, shown as

$$\Delta v_{jk} = \arctan2(\sin\Delta v_{jk}, \cos\Delta v_{jk}) + 2\pi N_{jk}, \tag{4}$$

where Δv_{jk} is the true anomaly differences, and the term of $2\pi N_{jk}$ is added under the calculation of the true anomaly differences in Algorithm 52. The second formula is the calculation of the semilatus rectum. For $\Delta v_{31} > 180^\circ$ always, the semilatus rectum p, can be written as

$$p = \frac{\sin\Delta v_{21} - \sin\Delta v_{31} + \sin\Delta v_{32}}{-\dfrac{\sin\Delta v_{31}}{|\boldsymbol{r}_2|} + \dfrac{\sin\Delta v_{21}}{|\boldsymbol{r}_3|} + \dfrac{\sin\Delta v_{32}}{|\boldsymbol{r}_1|}}, \tag{5}$$

where the denominator cannot equal to zero. The last formula is the calculation of the eccentric anomaly differences:

$$\Delta E_{jk} = \arctan2(\sin\Delta E_{jk}, \cos\Delta E_{jk}) + 2\pi N_{jk}, \tag{6}$$

where ΔE_{jk} is the eccentric anomaly differences, and the term of $2\pi N_{jk}$ is added under the calculation of the eccentric anomaly differences in Algorithm 52.

3 Numerical Simulation

Select three GEO objects: zero-inclination and low eccentricity, medium inclination and low eccentricity, and large inclination and zero-eccentricity, as the observation objects. Table 1 exhibits the orbital elements of the three satellites, where a is the semimajor axis, e is the eccentricity, i is the inclination, ω is the argument of perigee, Ω is the RAAN, and f is the true anomaly. The platform of an SBV sensor is a Sun-synchronous orbit satellite, whose orbital period is 97 min and the descending node is at 18:00 LT[12]. The right ascension and declination of the search fence center at 52° and 0°, respectively. The field of view of the SBV sensor is 4° × 4°, and the time span of an arc is about 30 s[13].

The ephemeris error of the surveillance sensor, the timing error between the surveillance satellite and surveillance object, the boresight pointing error of SBV sensor, and the error of streak endpoint determination are considered, when simulating the observation data[14]. Here the ephemeris error is 15 m, the timing error is 1 ms, and other errors are 2″.

Table 2 shows the observation times of the three arcs for GEO satellites. The observation time is described in seconds from the time when the simulation begins. In this table, the boldfaced observation times are used to determine the initial orbit. The days of

Table 1 Orbital elements of GEO satellites

Object	a/km	e	i/(°)	ω/(°)	Ω/(°)	f/(°)
G1	42 164.1696	0.05	0	0	338.73	0
G2	42 164.1696	0.05	8.0	0.0	52.0	46.0
G3	42 164.1696	0	15.00	0.00	52.0	46.0

Table 2 Observation times of the three arcs for GEO satellites

Objects	Arc1/s	Arc2/s	Arc3/s	days of arcs
G1	76 921-76 950	245941-245 970	333 181-333 210	(0/2/3)
G2	76 981-77 010	246001-246 030	251 461-251 490	(0/2/2)
G3	76 921-76 950	245941-245 970	420 361-420 390	(0/2/4)

the three arcs from the simulation begins are denoted as (Arc1/Arc2/Arc3). As seen in Table 2, the largest spreads in the three points is about 4 days.

Table 3 shows the differences between the results of IOD by improved double r-iteration method and the theoretical values. The position accuracy of the initial orbit is at kilometers level, and the velocity accuracy of the initial orbit is at the level of several meters per second.

4 Conclusions

In this paper, we introduce a specific number of complete revolutions for the three points into the double r-iteration method, to solve the IOD problem whose central angle subtended by the three points is much more than 360°. The feasibility and rationality of the improved double r-iteration method in the paper has been validated by the simulation experiments.

Table 3 Differences between the results of IOD by improved double r-iteration method and the theoretical values

items	G1	G2	G3
Δx/km	4.7121	−4.8693	1.9205
Δy/km	3.7875	−4.5135	−1.9019
Δz/km	−1.4896	0.9207	0.2457
$\Delta v_x/(\mathrm{m\cdot s^{-1}})$	−5.7017	1.8392	2.9281
$\Delta v_y/(\mathrm{m\cdot s^{-1}})$	−6.8007	2.3627	3.4776
$\Delta v_z/(\mathrm{m\cdot s^{-1}})$	1.4635	−0.4135	−0.5989

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